What does it mean to be a competent mathematics teacher?

A general problem illustrated by examples from Denmark

Mogens Niss, IMFUFA, Department of Science
Roskilde University, Denmark

Introduction

It is well known that delicate balances have to be struck and genuine dilemmas have to be resolved for the design, structuring, and organisation of pre-service education and in-service professional development of competent mathematics teachers. Let us phrase one of the typical dilemmas as follows: Is a competent mathematics teacher someone who has been trained in mathematics and receives some additional pedagogical or didactical training - of a theoretical and/or practical nature – and then moves on to teaching mathematics? Or is a competent mathematics teacher someone who has received pedagogical training and moves on to teaching mathematics after having had some amount of mathematical training? Another classical dilemma is to do with mathematical content: Should the mathematics teachers learn, be mathematics as such, without particular regard to their future teaching profession, or should they learn mathematics in ways that are specifically focused on teaching at certain levels, i.e. focus on what Shulman (1986) calls "pedagogical content knowledge"? Yet another issue is the balance between theoretical study in a teacher training institution and practical work in school.

These dilemmas and challenges are all genuine, in the sense that there are good arguments in favour of each of the options sketched, and both clear advantages and drawbacks attached to every one of them. Therefore any choice between them – or between other related options - comes at a cost. This is presumably the main reason why different places have adopted made different approaches to the design and structuring of teacher training programmes in mathematics, sometimes varying with the educational level served by a given programme. The present paper reviews these approaches taking is point of departure in a description and discussion of the situation in Denmark. The main point of the paper is to propose a new way to look at mathematics teachers' competencies in order to better link the different components in the preparation and professional development of mathematics teachers at all levels.

The education of mathematics teachers in Denmark.

A fundamental characteristic of the state of affairs in Denmark is that two completely different teacher education systems are in force.

Teachers for the primary and lower secondary levels, grades K-9/10, are educated in special teacher training colleges, which are not part of the university system and in which the staff have no research rights or duties. The teachers are prepared to work in the public – municipal – schools, called "folkeskoler" ("schools for the people"), which is why they are called "folkeskole" teachers in usual parlance. The far majority of the grade K-9/10 private schools recruit their teachers from the same teacher training colleges as do the public schools. The formal duration of the "folkeskole" teacher training programme is 3½ years, but many students spend 4 years to complete it. Organised – and compulsory - elements of school teaching practice form part of the pre-service training.

In contrast, teachers for the different kinds of upper secondary levels (grades 10-12) (and more advanced levels) are educated in universities and are required to have a (combined) master's degree in two subjects relevant for the school type in which they seek a position. In everyday parlance, for brevity these schools are called "gymnasiums", and their teacher are usually referred to as "gymnasium" teachers. The formal duration of a master's degree is 5 years, but for most students it takes 6 or 7 years to earn a degree. The "gymnasium" schools do not belong to the municipalities but to the state (from 1985 to 2006, they actually belonged to the counties). In addition to a master's degree, a prospective gymnasium teacher has to take a combined theoretical and practical course in pedagogy and didactics within a school setting, and under supervision of experienced "gymnasium" teachers. The main component is teaching practice, whereas the theoretical elements are few and of limited extension. The obtaining of a permanent position as a "gymnasium teacher" requires the candidate to have passed the combined theoretical and practical course. It is the candidate's supervisors who, together with external examiners (also experienced teachers), decide whether or not the candidate has passed.

In Denmark we are thus faced with a dual structure, both as regards schooling and as regards teacher preparation. Actually, there is a bit of an "iron curtain" between the two sub-systems. The "folkeskole" and the "folkeskole" teachers represent one closed sub-system, subjected to one politico-administrative system. If a "folkeskole" teacher wishes to change career and move to a "gymnasium", he or she would have to take a master's degree, even though (s)he might well be allowed to skip a few components in the university programme. The "gymnasiums" and their teachers represent another closed sub-system. It is worth noting that a "gymnasium" teacher, holding a master's degree in, say, mathematics, will not be allowed to teach in a "folkeskole" without special admission courses. The differences between the two sub-systems are not only to do with administrative, formal and structural issues, but are also to do with two different cultures, sets of ideologies, priorities etc.

So far, we have focused on certain general external aspects of the structure in Denmark. How about the internal structure and the issue of content? Let us concentrate on the mathematics teachers.

The programmes in the teacher training colleges are entirely focused on the fact the students will become teachers in the "folkeskole". General pedagogical and didactical components feature strongly in the programme. In addition, teacher students have to choose four school subjects for further specialisation, one of which can be mathematics. In principle, a "folkeskole" teacher who teaches mathematics should have chosen to specialise in mathematics in teacher training college. However as regards teachers who actually teach mathematics in the lower grades (K-4) 75% have *not* specialised in mathematics, and for the upper grades (7-9/10) as many as 25% of those who teach mathematics have no formal background in mathematics from their pre-service training.

Now, in the mathematics taught in the teacher training colleges, the teaching and learning of mathematics as such is integrated with the pedagogical and didactical aspects of mathematics. In other words, prospective "folkeskole" teachers only encounter mathematics as an educational subject – focused on "pedagogical content knowledge" - not as a subject in its own right. There are some advantages of this approach, such as: Firstly, the students have chosen to study for the profession of teacher, and hence have their attention focused on what it takes to be a "folkeskole" teacher and on preparing for that specific job. Secondly, specific attention is being paid, already in the pre-service studies, to pedagogical and didactical aspects of the teaching and learning of mathematics in "folkeskole" contexts. It is not left to the individual teacher him- or herself,

afterwards in school, to transform and transpose his or her mathematical knowledge into an educational subject. This implies that students are familiar, at least to some extent, with some of the kinds of problems, issues, challenges, possibilities and limitations they are to encounter when subsequently taking up the profession of mathematics teacher. This approach, however, also carries serious drawbacks with it. Firstly, the overall mathematical background of "folkeskole" teachers is, in fact, rather limited, so limited that this is not only a problem in itself, but also a problem for a proper substantiation of the didactical perspectives being part of their preparation. Simpl(isticall)y put, their mathematical "radius of action" is rather small. Secondly, the education of a "folkeskole" teacher is focused on the current situation of the "folkeskole" in general, and of mathematics in particular. If either becomes subject of marked changes (which is often the case in Denmark), teachers' mathematical (and pedagogo-didactical) preparation tends to become outdated. Content oriented in-service courses are needed to update or upgrade teachers. Thirdly, "folkeskole" teachers have only limited possibilities of helping their students succeed in the transition from "folkeskole" to "gymnasium", because they do not know and understand enough about the mathematical content and perspectives adopted at the "gymnasium" level.

In contrast, most of the university programmes in Denmark through which "gymnasium" teachers are trained are predominantly focused on mathematics as a discipline. As there are differences between the approaches taken by the different universities, some if them (including the authors' university) contain certain components dealing with the didactics of mathematics, but these components, if they exist, usually have a rather limited presence in the overall programme. In most places, students do not know, during their university studies, whether they are actually going to be "gymnasium" teachers. This is often a decision that ripens late during their studies, or even after their completion of a master's degree. Again, there are indeed advantages of this approach. Firstly, graduates are usually equipped with rather solid knowledge and experiences of mathematics in at least some of its manifestations as a discipline. This means that those who become teachers can bring a multi-faceted set of mathematical insights into their teaching. Secondly, the mathematical background of "gymnasium" teachers is usually robust enough to be feasible also under changed conditions, and it is rather easy for them to take new mathematical knowledge on board during their career. Thirdly, as they have a university background in mathematics, they are usually able to inform and advise their students as regards the transition from "gymnasium" to tertiary studies in which mathematics plays a serious role. But there are certainly drawbacks as well. The most important one is that even though prospective teachers will have to take a (rather short) theoretical course in general pedagogy and didactics of mathematics, and a practical course as a "teacher trainee", their pedagogical and didactical preparation is usually very modest, in particular as regards issues related to mathematical learning. Secondly, they have not been prepared for the structural and organisational conditions that are prevalent in the "gymnasium" schools. And thirdly, their knowledge of what happens in primary and lower secondary school mathematics teaching is often not so much rooted in updated insight into that part of reality as in their own personal school experiences of a fair number of years ago.

One of the consequences of the strong duality in the preparation of mathematics teachers in Denmark is a strong divide between "folkeskole" and "gymnasium" teachers' knowledge about, conceptions and views of, attitudes to, and pedagogico-didactical perspectives on mathematics, as well as on its teaching and learning. This has several unfortunate consequences, one of which is a problematic transition process for students leaving lower and entering upper secondary mathematics education. Therefore, there is a need to find a "third way" to educate competent mathematics teachers, in order to avoid some of the dilemmas just outlined.

What does it mean to be a competent mathematics teacher? A proposal for a new answer.

As part of a larger project in Denmark, the so-called KOM-project of which I was the director, a new approach to characterising what it means to master mathematics and to what it means to be a competent mathematics teacher, was developed. In this paper I shall focus on the latter issue, but in order to do so I have to begin by considering the former issue.

If we want to characterise what it means to master mathematics – to possess mathematical competence - in ways that are invariant across educational levels and across mathematical topics, we must first give a general definition and then identify the key constituents in mathematical competence.

To possess **mathematical competence** means having knowledge of, understanding, doing, using and having a well-founded opinion about mathematics in a variety of situations and contexts where mathematics plays or can play a role

In the KOM project we identified eight main constituents in that competence, each of which is called a mathematical competency. More specifically we define a mathematical competency as insight-based preparedness to act purposefully and efficiently in situations that contain *a certain type* of mathematical challenges. Without going into a detailed description and discussion of the individual competencies (see, e.g., Niss , 2003), the system consists of the following competencies:

The ability to ask and answer questions in and with mathematics:

Mathematical thinking competency – mastering mathematical modes of thought

Problem handling competency – formulating and solving mathematical problems

Modelling competency – being able to analyse and build mathematical models concerning other subjects or practice areas

Reasoning competency – being able to reason mathematically

The ability to deal with mathematical language and tools:

Representation competency – being able to handle different representations of mathematical entities

Symbol and formalism competency – being able to handle symbol language and formal mathematical systems

Communication competency – being able to communicate, in, with, and about mathematics

Aids and tools competence – being able to make use of and relate to the aids and tools of mathematics.

Linguistic competence (with respect to a specific language) is constituted by four competencies, the ability to read and decode different sorts of texts, the ability to listen to and decode different kinds of speech, the ability to speak in different genres so as to make oneself understood by various sorts of listeners, the ability to write different kinds of texts so as to make oneself understood by various sorts of readers. In the same way as linguistic competence is constituted by the same components from grade 1 through to the university level, but manifests itself very differently as regard the specific texts and sorts of speech at issue, mathematical competence consists of the same components – the competencies - at all levels, but plays out very differently with respect to the mathematical substance on which the competencies are brought to bear.

The relationship between mathematical competencies and mathematical subject matter is a two-dimensional one - they are "orthogonal" to one another. It follows that it is not possible to deduce which mathematical subject matter should be on the agenda in a given educational context from the competencies alone.

We are now able to characterise what it means to be a competent mathematics teacher. Well, the general part of the answer is easy: A competent mathematics teacher is someone who in an effective and efficient way is able to help his or her students build and develop mathematical competencies.

But what does that mean in more specific terms? Well, two things.

Firstly a competent mathematics teacher must be mathematically competent him- or herself, in the sense outlined above. That is, a competent mathematics teacher must possess the eight competencies to a degree relevant for the educational level on which his / her teaching takes place. It follows that as regards the mathematical pre-service education of the teacher it is not sufficient to provide it to in such a way that the study of mathematics is restricted to the study of mathematics in its didactical and pedagogical enactment as en educational subject.

Secondly, a competent mathematics teacher must possess six didactical and pedagogical competencies with particular regard to mathematics:

- **Curriculum competency**:

To analyse, assess, relate to, and implement existing mathematics curricula and syllabi, and to construct new ones

- **Teaching competency**:

To devise, plan, organise, orchestrate and carry out mathematics teaching, including: creating a rich spectrum of teaching/learning situations; find, assess, select and create teaching materials; inspire and motivate students; discuss curricula and justify teaching/learning activities with students.

- **Uncovering of learning competency**:

To uncover, interpret and analyse students' learning of mathematics, as well as their notions, beliefs and attitudes towards mathematics. Includes identifying development with the individual student

- **Assessment competency**:

To identify, assess, characterise, and communicate students' learning outcomes and competencies, so as to inform and assist the individual student, and other relevant parties. This includes selecting,

modifying, constructing, critically analysing, and implementing a varied set of assessment forms and instruments to serve different formative and summative purposes.

All of these have to be geared toward different categories of recipients in different situations, and at different levels, paying attention to the individual student's needs and opportunities

- **Collaboration competency**:

To collaborate with different sorts of colleagues in and outside mathematics, as well as others (parents, authorities) concerning mathematics teaching and its conditions

- **Professional development competency**:

To develop one's own competency as a mathematics teacher (a meta-competency), including participate in and relate to activities of professional development, such as in-service courses, projects, conferences; reflect upon one's own teaching and needs for development; keep oneself up-dated about new developments and trends in research and practice.

It follows that for someone to be a competent mathematics teacher it is not sufficient to be well educated in mathematics as a discipline and then be equipped with a limited amount of theoretical pedagogy and didactics of mathematics plus practical training. Didactics of mathematics must be a crucial component in the pre-service education of a future mathematics teacher, in particular as far as the first four of these competencies are concerned. The two last-mentioned competencies must be developed in service as part of professional development.

Conclusion

This way of looking at mathematics teachers' competencies and their development during pre-service education and in in-service professional development activities poses substantial challenges to the Danish educational system, both in terms of structure and organisation, and in terms of human and material resources. It would not be surprising if this were true of other countries as well.

References

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